Memorandum on the relation between MC weight, the effective area, and the sensitivity

Shigeru Yoshida

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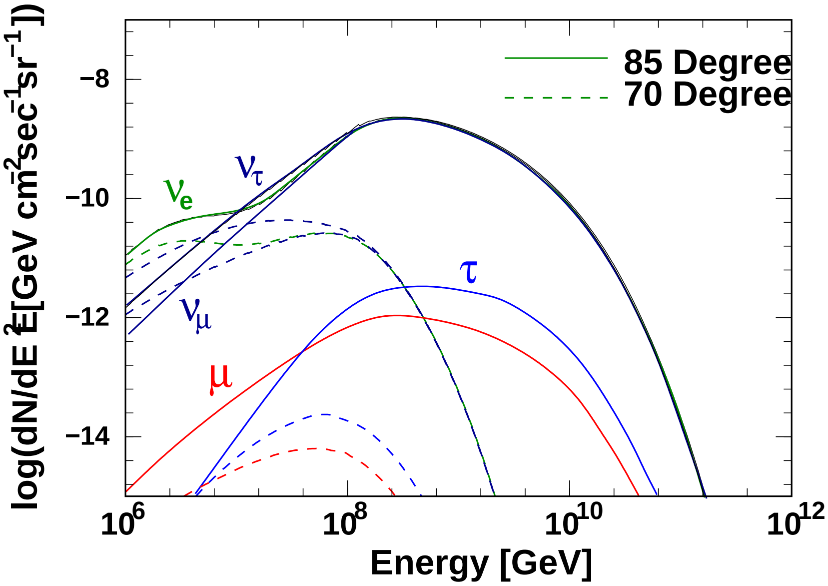

Figure 1: Fluxes of the EHE particles at the IceCube depth for a scenario of the neutrino production by the GZK mechanism. Two cases in the nadir angle are shown in the figure.

Expected event rate N_{ev} from a primary neutrino flux J_ν at the earth surface can be obtained by

$$\frac{dN_{ev}}{d\log E_l d\Omega} = A^{eff}(\log E_l, \Omega) \int_{\log E_l} d\log E_\nu \frac{dN_{\nu\to l}}{d\log E_l}(\log E_\nu, \log E_l, \Omega)\frac{dJ_\nu}{d\log E_\nu d\Omega}(\log E_\nu, \Omega), \tag{1}$$

where E_l is energy of the secondary lepton such as μ, E_ν is primary energy of neutrinos, $N_{\nu\to l}$ is number of secondary leptons produced inside the earth and reaching to the IceCube volume, and A^{eff} is the effective area of the IceCube observatory. The integral in the equation above accounts for the propagation effect in the earth and obtained by resolving the relevant transport equation [1]. This is equivalent to the secondary lepton flux at the IceCube depth and pre-calculated by JULIeT class like PropagatingNeutrinoFlux.java or PropagatingAtmMuonFlux.java. An example is shown in Figure 1.

The effective area A^{eff} in Eq. 1 can be estimated by either the semi-analytical way [1] or the full-brown MC. The semi-analytical method gives A^{eff} as

$$A^{eff}(\log E_l, \Omega) = \int_{\log E_{dep}^{th} \simeq 10\mathrm{PeV}} d\log E_{dep} \frac{dN_{dep}}{d\log E_{dep}}(\log E_l, \log E_{dep}, \Omega) A_0 \quad (2)$$

where E_{dep} is energy deposit of the secondary lepton propagating over 1km inside the IceCube volume. In EHE the energy deposit takes place mostly in form of a bunch of cascades. A_0 is typically 1 km^2 for the IceCube.

By the full MC, the effective area will be given by

$$\begin{aligned} A^{eff}(\log E_l, \Omega) &= A_0 \frac{N^{detected}}{\frac{dN_l^{MC}}{d\log E_l}(\log E_l, \Omega)\Delta \log E_l} \\ &= A_0 \sum^{detected} \frac{1}{\frac{dN_l^{MC}}{d\log E_l}(\log E_l, \Omega)\Delta \log E_l}, \end{aligned} \quad (3)$$

where $N^{detected}$ is number of MC events passing your criteria, $dN_l^{MC}/d\log E_l$ is the MC primary particle spectrum of the secondary leptons (mostly μ and τ for EHE). A_0 is the area of throwing primary particles in MC. $\Delta \log E_l$ is a bin width. Putting Equations [1] and [3] together will lead to the proper event weight as

$$w = A_0 \frac{1}{\frac{dN_l^{MC}}{d\log E_l}(\log E_l, \Omega)\Delta \log E_l} \int_{\log E_l} d\log E_\nu \frac{dN_{\nu \to l}}{d\log E_l}(\log E_\nu, \log E_l, \Omega) \frac{dJ_\nu}{d\log E_\nu d\Omega}. \quad (4)$$

Note that J_ν could be given by the GZK neutrino model, or any other EHE neutrino models as well as by the atmospheric muon/neutrino background prediction. So the weight given above could be not unique but various depending on what models you consider.

In this context the IceCube "sensitivity" for EHE neutrinos can be obtained from a quasi-differential event rate in neutrino model independent way. This approach has been widely used in many other experiments (for example, see [2]). The neutrino flux upper-bound with energy of E_0 from non-existence of signals is evaluated by putting

$$\frac{dJ_\nu}{d\log E_\nu d\Omega}(\log E_\nu, \Omega) = \left[\frac{dJ_\nu}{d\log E_\nu d\Omega}(\log E_0, \Omega)\right] \delta(\log E_\nu - \log E_0) \quad (5)$$

into Eq. 1. We get

$$\left[\frac{dJ_\nu}{d\log E_\nu d\Omega}(\log E_0,\Omega)\right] = \frac{N^{ev}(\geq \log E_l^{th})^{95\%}}{\int d\Omega \int\limits_{\log E_l^{th}} d\log E_l A^{eff}(\log E_l,\Omega)\frac{dN_{\nu\to l}}{d\log E_l}(\log E_0,\log E_l,\Omega)} \tag{6}$$

where E_l^{th} is threshold energy of the secondary leptons, $N^{ev}(\geq \log E_l^{th})^{95\%}$ is number of upper bound events with 95 % C.L. The Poisson distribution gives 2.3 events for example.

Bibliography

[1] S. Yoshida, R. Ishibashi, H. Miyamoto Phys. Rev. D **69** 103004 (2004).

[2] X. Bertou et.al., Astroparticle Physics **17** 183-193 (2002).